# $GW_0$-Eliashberg Theory with Fourier Convolutions via Intermediate Representation

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Quantitative calculation for phonon-mediated superconductors including Coulomb interaction, though being a classic subject, has significantly advanced recently thanks to modern computational power and algorithms. Complete inclusion of the wavenumber and frequency dependences has remained challenging due to the difficulty in treating multiple energy scales: temperature, phonon and electron bandwidths. We develop a method to solve the Eliashberg equations with wavenumber- and frequency-dependent $GW_0$ self-energy corrections by phonon-mediated attraction and screened Coulomb repulsion. Tedious convolutions in Matsubara frequency and wavenumber are efficiently performed with the Fourier convolution theorem. The intermediate representation and sparse sampling [1,2] significantly reduce the number of quadrature points from $O(\beta)$ to $O(\log \beta)$, where $\beta$ denotes the inverse temperature.

We apply the method to the uniform electron plus Einstein phonon model to clarify how the Eliashberg equations including the $GW_0$-screened Coulomb self-energy behaves in an idealized metallic limit [2,3]. The dynamical screening effect on the superconducting Tc is revealed to vary through a range of electron density from negative to positive, which quantifies the plasmon-mediated superconducting pairing effect [4]. Thorough comparison with the static-Coulomb Eliashberg calculations [5] and superconducting density functional theory [6,7,8] is also presented.

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